IEEE WCNC: « Aggregation of Multi-Armed Bandits Learning Algorithms for Opportunistic Spectrum Access »

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CentraleSupélec IETR INSTITUT D'ÉLECTRONIQUE ET DE TÉLÉCOMMUNICATIONS DE RENNES	Inria INVENTEURS DU MONDE NUMÉRIQUE

> See our paper `HAL.Inria.fr/hal-01705292`

Introduction

- Cognitive Radio (CR) is known for being one of
  the possible solution to tackle the spectrum scarcity issue
- Opportunistic Spectrum Access (OSA) is a good model
  for CR problems in **licensed bands**
- Online learning strategies, mainly using multi-armed bandits (MAB) algorithms, were recently proved to be efficient `[Jouini 2010]`
- 💥 But there is many different MAB algorithms…
  which one should you choose in practice?

$\Longrightarrow$ we propose to use an online learning algorithm to also decide
which algorithm to use, to be more robust and adaptive to unknown environments.

⏲ Outline [15 min]

1. Opportunistic Spectrum Access [3 min]
2. Multi-Armed Bandits [2 min]
3. MAB algorithms [3 min]
4. Aggregation of MAB algorithms [3 min]
5. Illustration [3 min]

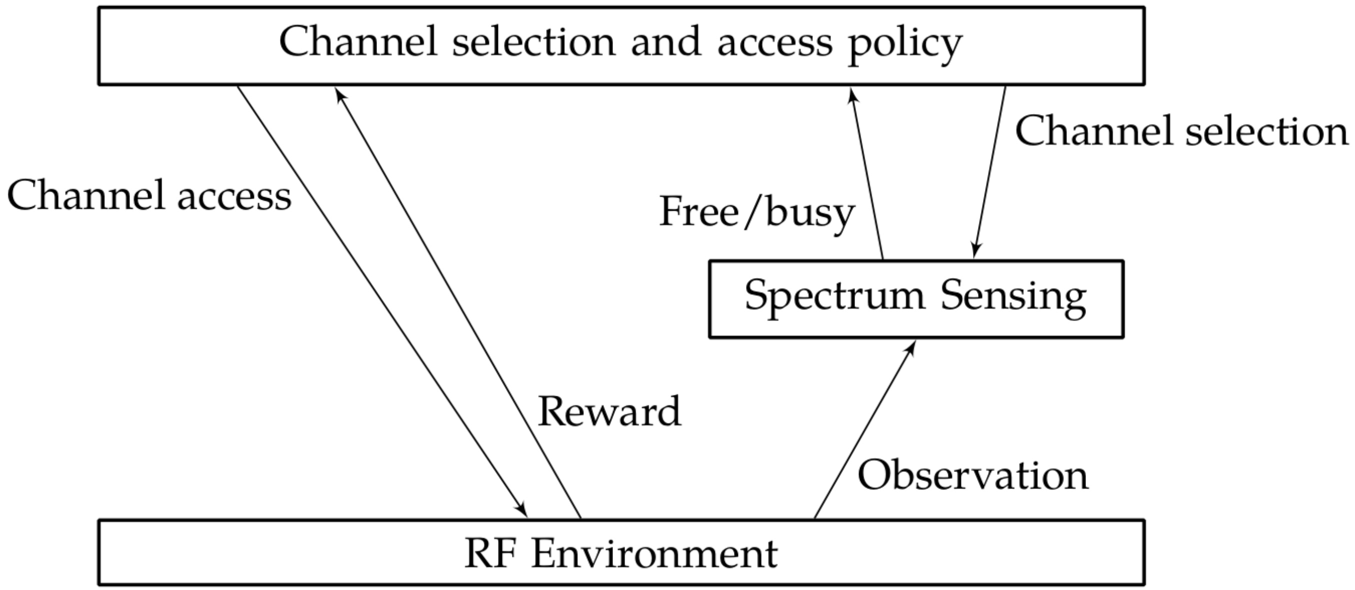
Please 🙏

Ask questions *at the end* if you want!

1. Opportunistic Spectrum Access [3 min]

- Spectrum scarcity is a well-known problem
- Different range of solutions…
- Cognitive Radio is one of them
- Opportunistic Spectrum Access is a kind of cognitive radio

Communication & interaction model



- 📱 Primary users are occupying K radio channels
- ☎️ Secondary users can sense and exploit free channels: want to **explore** the channels, and learn to **exploit** the best one
- Discrete time for everything $t \geq 1, t \in \mathbb{N}$

2. Multi-Armed Bandits [2 min]

Model

- Again $K \geq 2$ resources (*e.g.*, channels), called **arms**
- Each time slot $t = 1, \dots, T$, you must choose one arm, denoted $A(t) \in \{1, \dots, K\}$
- You receive some reward $r(t) \sim \nu_k$ when playing $k = A(t)$
- **Goal:** maximize your sum reward $\sum_{t=1}^{T} r(t)$
- Hypothesis: rewards are stochastic, of mean μ_k. *E.g.*, Bernoulli

Why is it famous?

Simple but good model for **exploration/exploitation** dilemma.

3. MAB algorithms [3 min]

- Main idea: index $I_k(t)$ to approximate the quality of each arm k
- First example: *UCB algorithm*
- Second example: *Thompson Sampling*

3.1 Multi-Armed Bandit algorithms

Often *index* based

- Keep *index* $I_k(t) \in \mathbb{R}$ for each arm $k = 1, \dots, K$
- Always play $A(t) = \arg\max I_k(t)$
- $I_k(t)$ should represent our belief of the *quality* of arm k at time t

Example: "Follow the Leader"

- $X_k(t) := \sum_{s<t} r(s) \mathbf{1}(A(s) = k)$ sum reward from arm k
- $N_k(t) := \sum_{s<t} \mathbf{1}(A(s) = k)$ number of samples of arm k
- And use $I_k(t) = \hat{\mu}_k(t) := \frac{X_k(t)}{N_k(t)}$.

3.2 First example of algorithm
Upper Confidence Bounds algorithm (UCB)

- Instead of using $I_k(t) = \frac{X_k(t)}{N_k(t)}$, add an exploration term

$$I_k(t) = \frac{X_k(t)}{N_k(t)} + \sqrt{\frac{\alpha \log(t)}{2N_k(t)}}$$

Parameter α: tradeoff exploration *vs* exploitation

- Small α: focus more on **exploitation**
- Large α: focus more on **exploration**

💥 **Problem: how to choose "the good α" for a certain problem?**

3.3 Second example of algorithm *Thompson sampling* (TS)

- Choose an initial belief on μ_k (uniform) and a prior p^t (*e.g.*, a Beta prior on $[0, 1]$)
- At each time, update the prior p^{t+1} from p^t using Bayes theorem
- And use $I_k(t) \sim p^t$ as *random* index

Example with Beta prior, for binary rewards

- $p^t = \mathrm{Beta}(1 + \text{nb successes}, 1 + \text{nb failures})$.
- Mean of $p^t = \frac{1+X_k(t)}{2+N_k(t)} \simeq \hat{\mu}_k(t)$.

💥 **How to choose "the good prior" for a certain problem?**

4. Aggregation of MAB algorithms [3 min]

Problem

- How to choose which algorithm to use?
- But also… Why commit to one only algorithm?

Solutions

- Offline benchmarks?
- Or online selections from a pool of algorithms?

$\hookrightarrow$ Aggregation?

> Not a new idea, studied from the 90's in the ML community.

- Also use online learning to *select the best algorithm*!

4.1 Basic idea for online aggregation

If you have $\mathcal{A}_1, \dots, \mathcal{A}_N$ different algorithms

- At time $t = 0$, start with a uniform distribution π^0 on $\{1, \dots, N\}$ (to represent the **trust** in each algorithm)
- At time t, choose $a^t \sim \pi^t$, then play with $\mathcal{A}_{a^t}$
- Compute next distribution π^{t+1} from π^t:
  - increase $\pi^{t+1}_{a^t}$ if choosing $\mathcal{A}_{a^t}$ gave a good reward
  - or decrease it otherwise

Problems

1. How to increase $\pi^{t+1}_{a^t}$?
2. What information should we give to which algorithms?

4.2 Overview of the *Exp4* aggregation algorithm

> For rewards in $r(t) \in [-1, 1]$.

- Use π^t to choose randomly the algorithm to trust, $a^t \sim \pi^t$
- Play its decision, $A_{\mathrm{aggr}}(t) = A_{a^t}(t)$, receive reward $r(t)$
- And give feedback of observed reward $r(t)$ only to this one
- Increase or decrease $\pi^t_{a^t}$ using an exponential weight:

$$\pi^{t+1}_{a^t} := \pi^t_{a^t} \times \exp\left(\eta_t \times \frac{r(t)}{\pi^t_{a^t}}\right).$$

- Renormalize π^{t+1} to keep a distribution on $\{1, \dots, N\}$
- Use a sequence of decreasing *learning rate* $\eta_t = \frac{\log(N)}{t \times K}$ (cooling scheme, $\eta_t \to 0$ for $t \to \infty$)

Use an *unbiased* estimate of the rewards

Using directly $r(t)$ to update trust probability yields a biased estimator

- So we use instead $\hat{r}(t) = r(t)/\pi_a^t$ if we trusted algorithm $\mathcal{A}_a$
- This way

$$\mathbb{E}[\hat{r}(t)] = \sum_{a=1}^{N} \mathbb{P}(a^t = a)\mathbb{E}[r(t)/\pi_a^t]$$

$$= \mathbb{E}[r(t)] \sum_{a=1}^{N} \frac{\mathbb{P}(a^t = a)}{\pi_a^t} = \mathbb{E}[r(t)]$$

4.3 Our *Aggregator* aggregation algorithm

Improves on *Exp4* by the following ideas:

- First let each algorithm vote for its decision $A^t_1, \dots, A^t_N$
- Choose arm $A_{\text{aggr}}(t) \sim p^{t+1}_j := \sum_{a=1}^{N} \pi^t_a \mathbf{1}(A^t_a = j)$
- Update trust for each of the trusted algorithm, not only one (*i.e.*, if $A^t_a = A^t_{\text{aggr}}$)
  $\hookrightarrow$ faster convergence
- Give feedback of reward $r(t)$ to *each* algorithm!
  (and not only the one trusted at time t)
  $\hookrightarrow$ each algorithm have more data to learn from

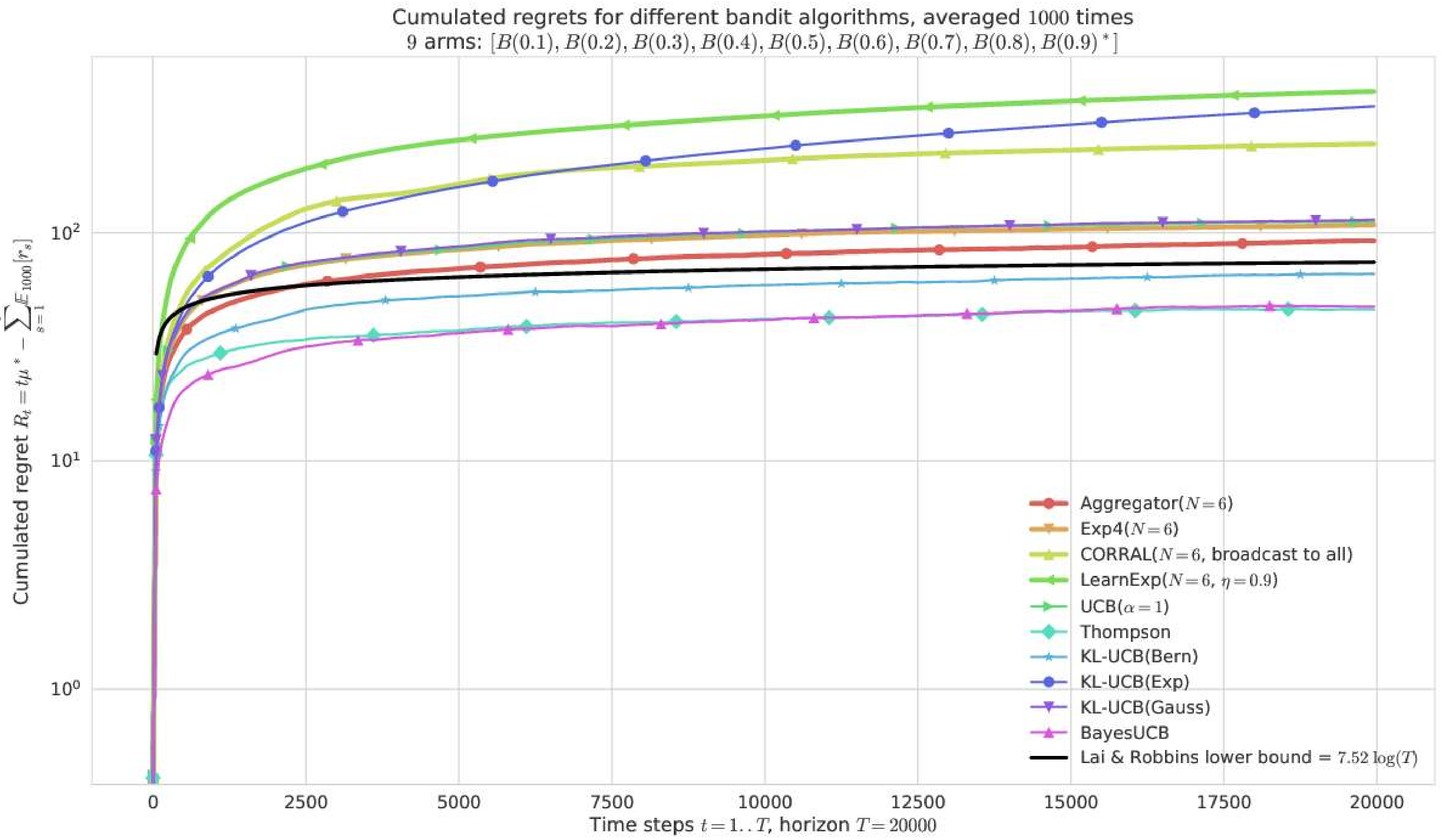
5. Some illustrations [3 min]

- Artificial simulations of stochastic bandit problems
- Bernoulli bandits but not only
- Pool of different algorithms (UCB, Thompson Sampling etc)
- Compared with other state-of-the-art algorithms for *expert aggregation* (Exp4, CORRAL, LearnExp)
- What is plotted it the *regret* for problem of means $\mu_1, \dots, \mu_K$:

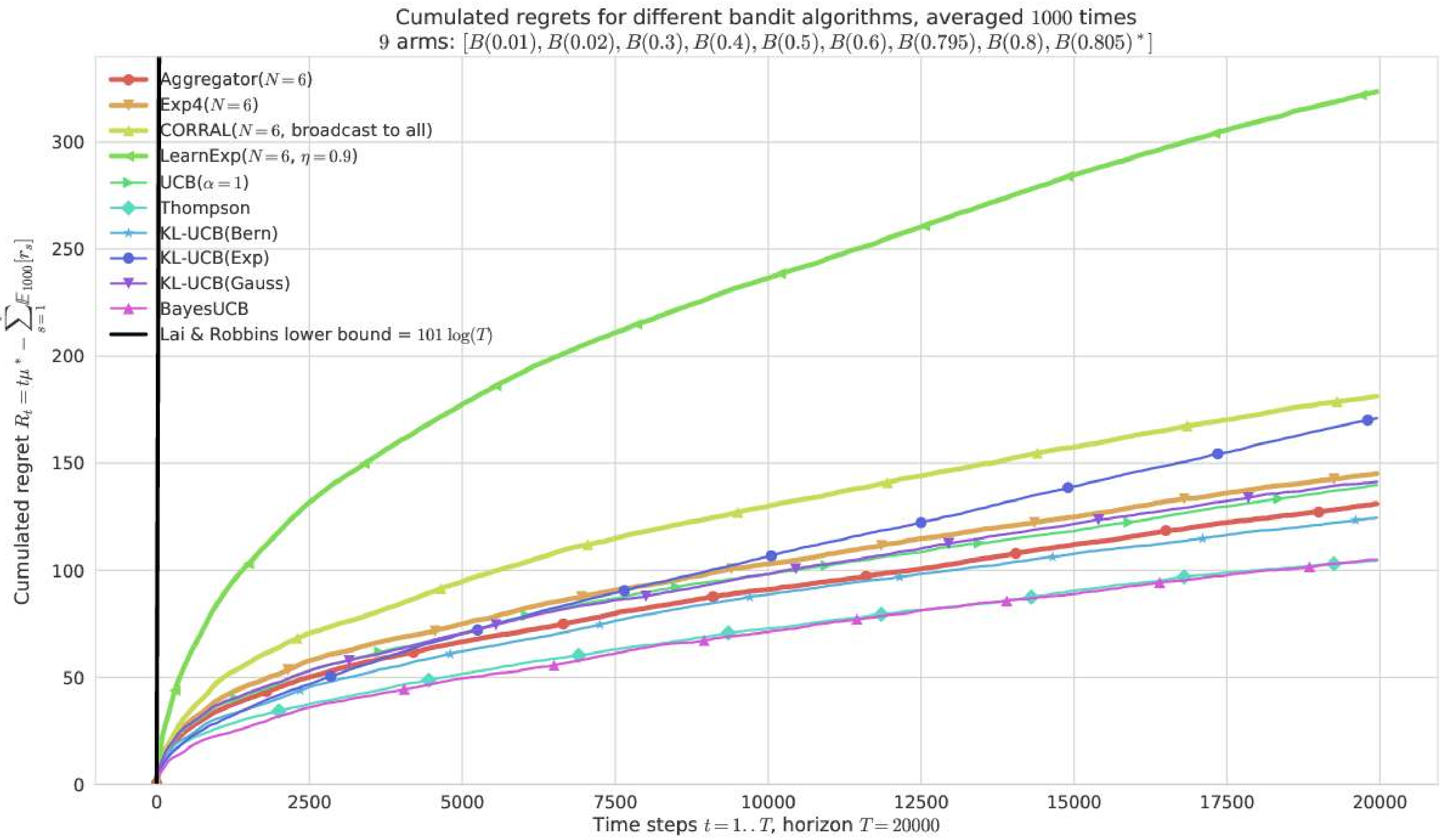
$$R_T^{\mu}(\mathcal{A}) = \max_k (T \mu_k) - \sum_{t=1}^{T} \mathbb{E}[r(t)]$$

- Regret is known to be lower-bounded by $C(\mu) \log(T)$
- and upper-bounded by $C'(\mu) \log(T)$ for efficient algorithms

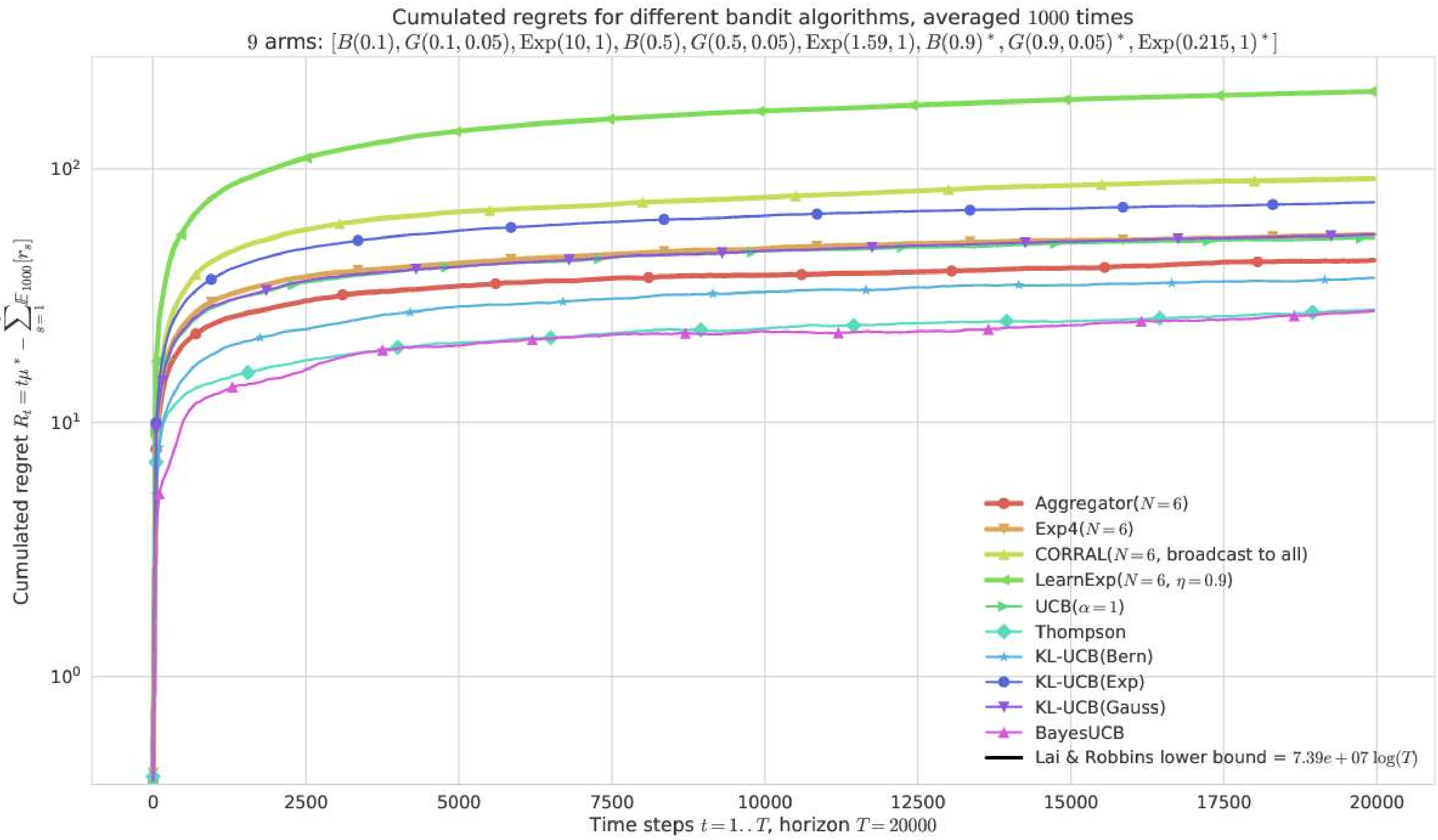
On a simple Bernoulli problem



On a "hard" Bernoulli problem



On a mixed problem



Conclusion (1/2)

- Online learning can be a powerful tool for Cognitive Radio, and many other real-world applications
- Many formulation exist, a simple one is the Multi-Armed Bandit
- Many algorithms exist, to tackle different situations
- It's hard to know before hand which algorithm is efficient for a certain problem…
- Online learning can also be used to select *on the run*
  which algorithm to prefer, for a specific situation!

Conclusion (2/2)

- Our algorithm **Aggregator** is efficient and easy to implement
- For N algorithms $\mathcal{A}_1, \dots, \mathcal{A}_N$, it costs $\mathcal{O}(N)$ memory, and $\mathcal{O}(N)$ extra computation time at each time step
- For stochastic bandit problem, it outperforms empirically the other state-of-the-arts (Exp4, CORRAL, LearnExp).

See our paper

`HAL.Inria.fr/hal-01705292`

See our code for experimenting with bandit algorithms

Python library, open source at `SMPyBandits.GitHub.io`

Thanks for listening !